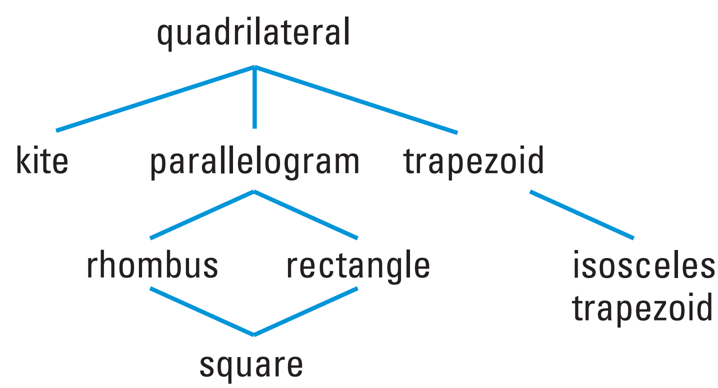
I can identify special quadrilaterals based upon limited information.
I can prove that a quadrilateral is a special type of quadrilateral.

CONCEPT SUMMARY — PROVING QUADRILATERALS ARE RHOMBUSES

You have learned three ways to prove that a quadrilateral is a rhombus.

1. You can use the definition and show that the quadrilateral is a *parallelogram* that has four congruent sides. It is easier, however, to use the Rhombus Corollary and simply show that all four sides of the quadrilateral are congruent.
2. Show that the quadrilateral is a parallelogram *and* that the diagonals are perpendicular. (*Theorem 6.11*)
3. Show that the quadrilateral is a parallelogram *and* that each diagonal bisects a pair of opposite angles. (*Theorem 6.12*)

In this chapter, you have studied the seven special types of quadrilaterals at the right. Notice that each shape has all the properties of the shapes linked above it. For instance, squares have the properties of rhombuses, rectangles, parallelograms, and quadrilaterals.

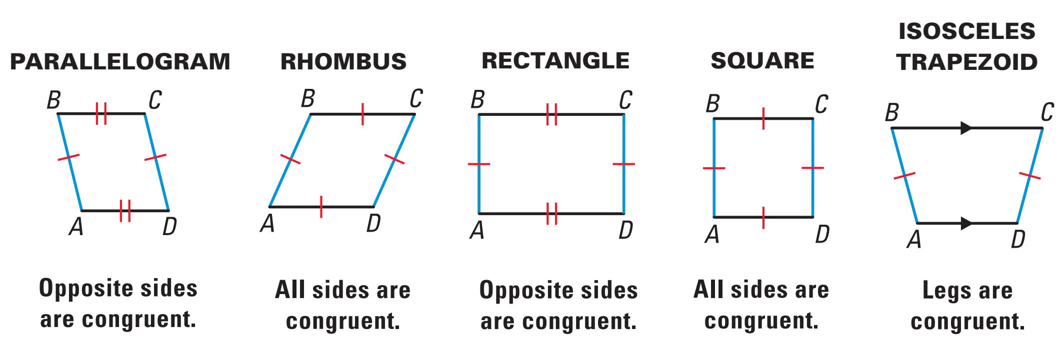


EXAMPLE 1 *Identifying Quadrilaterals*

Quadrilateral *ABCD* has at least one pair of opposite sides congruent. What kinds of quadrilaterals meet this condition?

SOLUTION

There are many possibilities.



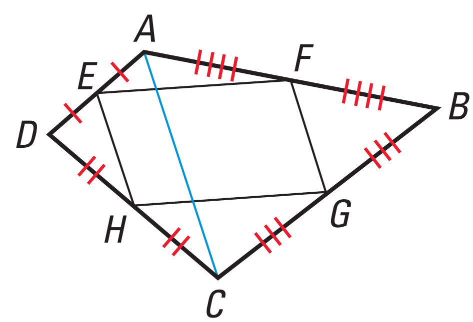
1. *ABCD* has at least two congruent consecutive sides. What quadrilaterals meet this condition?

EXAMPLE 2 *Connecting Midpoints of Sides*

When you join the midpoints of the sides of any quadrilateral, what special quadrilateral is formed? Why?

SOLUTION

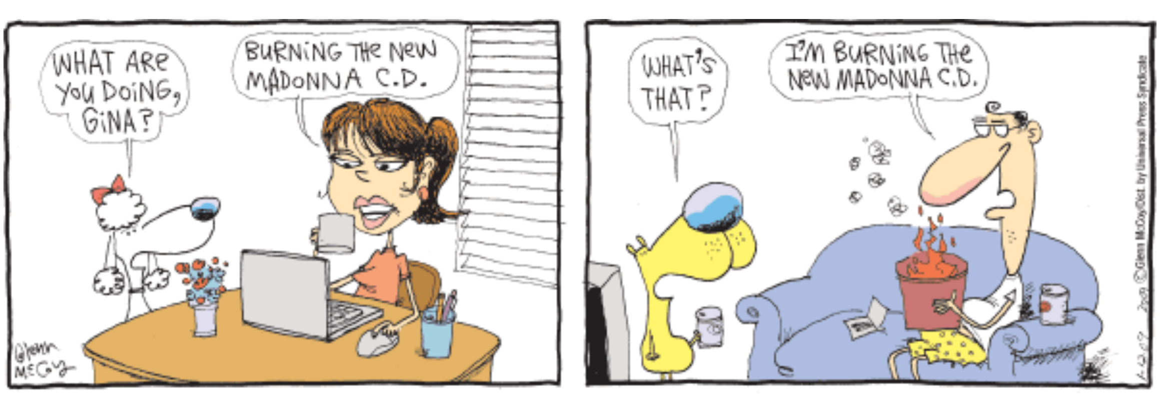
Let E, F, G, and H be the midpoints of the sides of any quadrilateral, $ABCD$, as shown.



If you draw $\overline{AC}$, the Midsegment Theorem for triangles says $\overline{FG} \parallel \overline{AC}$ and $\overline{EH} \parallel \overline{AC}$, so $\overline{FG} \parallel \overline{EH}$. Similar reasoning shows that $\overline{EF} \parallel \overline{HG}$.

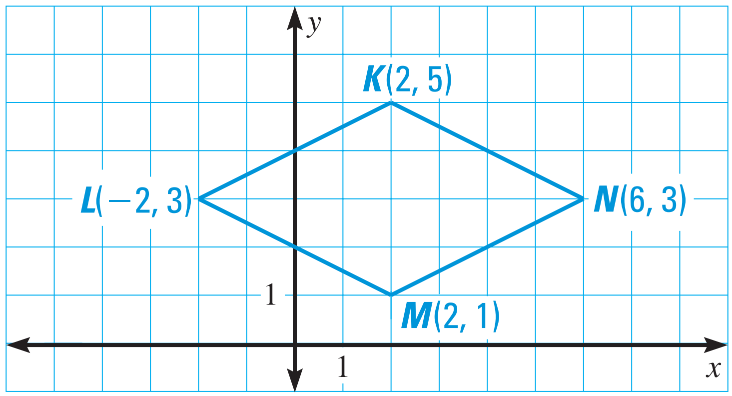
▶ So, by definition, *EFGH* is a parallelogram.

2. When you join the midpoints of the sides of an isosceles trapezoid in order, what special quadrilateral is formed? Why?



EXAMPLE 3 *Proving a Quadrilateral is a Rhombus*

Show that *KLMN* is a rhombus.



SOLUTION You can use any of the three ways described in the concept summary above. For instance, you could show that opposite sides have the same slope and that the diagonals are perpendicular. Another way, shown below, is to prove that all four sides have the same length.

$$LM = \sqrt{[2 - (-2)]^2 + (1 - 3)^2} = \sqrt{4^2 + (-2)^2} = \sqrt{20}$$

$$NK = \sqrt{(2 - 6)^2 + (5 - 3)^2} = \sqrt{(-4)^2 + 2^2} = \sqrt{20}$$

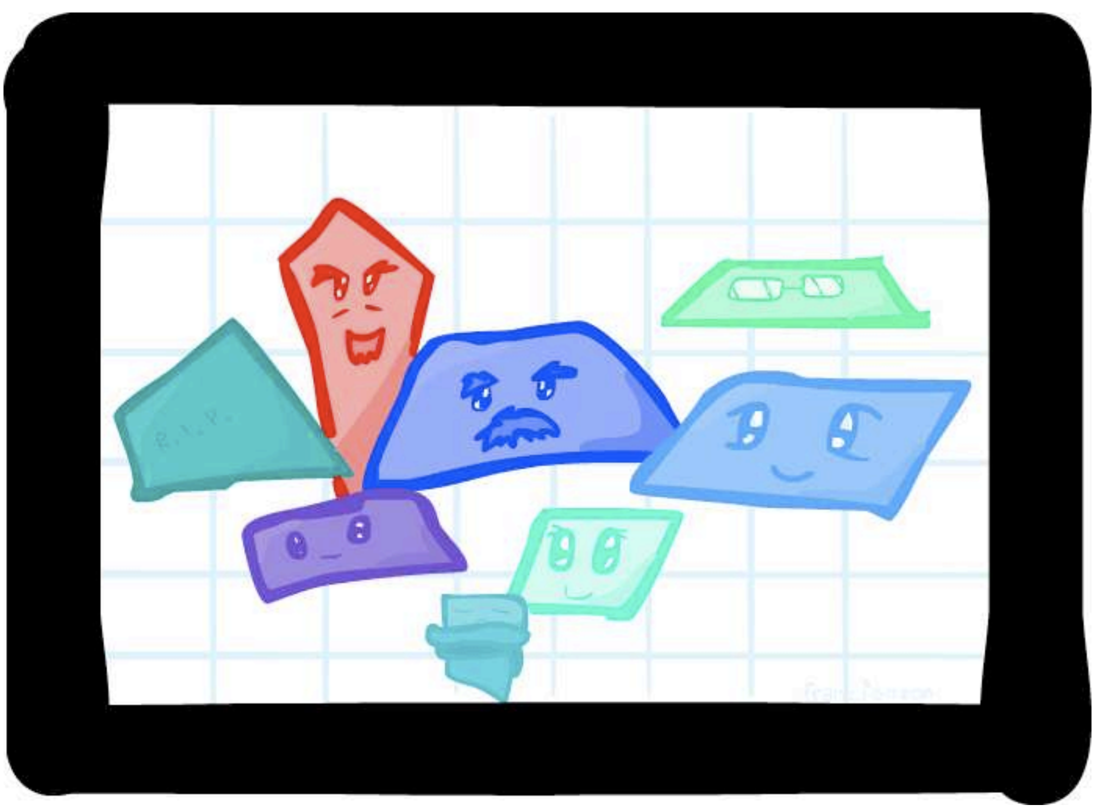
$$MN = \sqrt{(6 - 2)^2 + (3 - 1)^2} = \sqrt{4^2 + 2^2} = \sqrt{20}$$

$$KL = \sqrt{(-2 - 2)^2 + (3 - 5)^2} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{20}$$

▶ So, because $LM = NK = MN = KL$, *KLMN* is a rhombus.

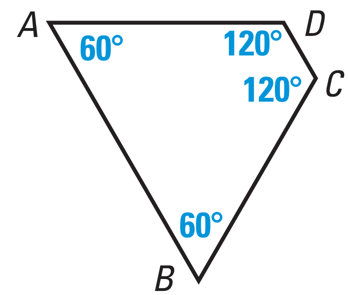
3. The coordinates of ABCD are A(-2, 5), B(1, 8) C(4, 5) & D(1, 2). Show that ABCD is a rhombus.

4. The diagonals of *RSTQ* are perpendicular. What quadrilaterals meet this condition?



EXAMPLE 4 *Identifying a Quadrilateral*

What type of quadrilateral is *ABCD*?
Explain your reasoning.



SOLUTION

$\angle A$ and $\angle D$ are supplementary, but $\angle A$ and $\angle B$ are not. So, $\overline{AB} \parallel \overline{DC}$ but $\overline{AD}$ is not parallel to $\overline{BC}$. By definition, *ABCD* is a trapezoid. Because base angles are congruent, *ABCD* is an isosceles trapezoid.

5. In quadrilateral RSTV, $m\measuredangle R = 88°$, $m\measuredangle S = 113°$, & $m\measuredangle V = 113°$. What type of quadrilateral could RSTV be? Must it be this type?

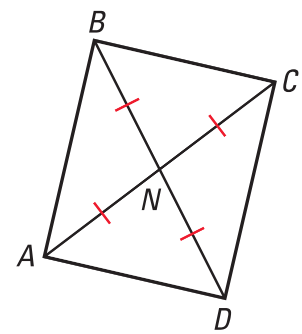
"… And remember: Rebounds! We need rebounds!!"

EXAMPLE 5 *Identifying a Quadrilateral*

The diagonals of quadrilateral $ABCD$ intersect at point N to produce four congruent segments: $\overline{AN} \cong \overline{BN} \cong \overline{CN} \cong \overline{DN}$. What type of quadrilateral is $ABCD$? Prove that your answer is correct.

SOLUTION

Draw a diagram:



Draw the diagonals as described. Then connect the endpoints to draw quadrilateral $ABCD$.

Make a conjecture:

Quadrilateral $ABCD$ looks like a rectangle.

Proof

Prove your conjecture:

GIVEN ▶ $\overline{AN} \cong \overline{BN} \cong \overline{CN} \cong \overline{DN}$

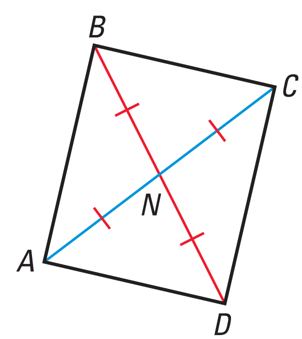
PROVE ▶ $ABCD$ is a rectangle.

Paragraph Proof Because you are given information about the diagonals, show that $ABCD$ is a parallelogram with congruent diagonals.

First prove that $ABCD$ is a parallelogram.

Because $\overline{BN} \cong \overline{DN}$ and $\overline{AN} \cong \overline{CN}$, $\overline{BD}$ and $\overline{AC}$ bisect each other. Because the diagonals of $ABCD$ bisect each other, $ABCD$ is a parallelogram.

Then prove that the diagonals of $ABCD$ are congruent.



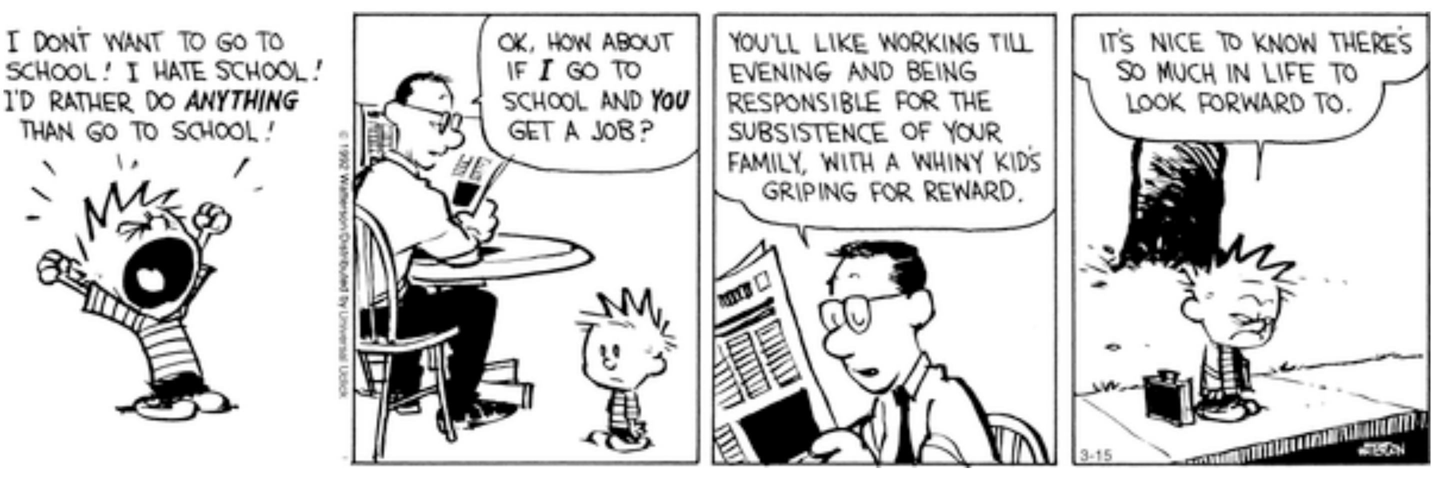
From the given you can write $BN = AN$ and $DN = CN$ so, by the Addition Property of Equality, $BN + DN = AN + CN$. By the Segment Addition Postulate, $BD = BN + DN$ and $AC = AN + CN$ so, by substitution, $BD = AC$.

So, $\overline{BD} \cong \overline{AC}$.

▶ $ABCD$ is a parallelogram with congruent diagonals, so $ABCD$ is a rectangle.

6. The diagonals of ABCD intersect to form four congruent isosceles triangles: $\triangle ANB \cong \triangle CNB \cong \triangle CND \cong \triangle AND.$ What type of quadrilateral is ABCD? Prove that the answer is correct.

7. The diagonals of ABCD are congruent. $m\measuredangle A = m\measuredangle B = 95° \;\&\; m\measuredangle C = m\measuredangle D = 85°.$ What kind of figure is ABCD?



8. Put an X in the box if the shape *always* has the given property.

Property	▱	Rectangle	Rhombus	Square	Kite	Trapezoid
Both pairs of opposite sides are ∥.						
Exactly one pair of opposite sides are ∥.						
Diagonals are $\perp$.						
Diagonals are $\cong$.						
Diagonals bisect each other.